UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

9709 MATHEMATICS

9709/32 Paper 32, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a product or quotient and remove logarithms	M1	
		Obtain quadratic equation $x^2 - 5x + 5 = 0$, or equivalent	A1	
		Solve 3-term quadratic obtaining 1 or 2 roots	A1	
		Obtain answers 1.38 and 3.62	A1	[4]
2	**(i)**	Evaluate, or consider the sign of, $x^3 - 8x - 13$ for two integer values of x, or equivalent	M1	
		Conclude $x = 3$ and $x = 4$ with no errors seen	A1	[2]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.43	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (3.425, 3.435)	A1	[3]
3	**(i)**	State $2xy + x^2 \dfrac{dy}{dx}$ as derivative of x^2y	B1	
		State $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	B1	
		Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	M1	
		Obtain answer $\dfrac{3x^2 - 2xy}{x^2 + 3y^2}$, or equivalent	A1	[4]
	(ii)	Find gradient of tangent at (2, 1) and form equation of tangent	M1	
		Obtain answer $8x - 7y - 9 = 0$, or equivalent	A1√	[2]
4		Use $\tan(A \pm B)$ formula and obtain an equation in $\tan\alpha$ and $\tan\beta$	M1*	
		Substitute throughout for $\tan\alpha$ or for $\tan\beta$	M1(dep*)	
		Obtain $2\tan^2\beta + \tan\beta - 1 = 0$ or $\tan^2\alpha + \tan\alpha - 2 = 0$, or equivalent	A1	
		Solve a 3-term quadratic and find an angle	M1	
		Obtain answer $\alpha = 45°$, $\beta = 26.6°$	A1	
		Obtain answer $\alpha = 116.6°$, $\beta = 135°$	A1	[6]
		[Treat answers given in radians as a misread. Ignore answers outside the given range.]		
		[SR: Two correct values of α (or β) score A1; then A1 for both correct α, β pairs]		
5	**(i)**	Substitute $x = -2$, equate to zero and state a correct equation, e.g. $-16 + 4a - 2b - 4 = 0$	B1	
		Differentiate p(x), substitute $x = -2$ and equate to zero	M1	
		Obtain a correct equation, e.g. $24 - 4a + b = 0$	A1	
		Solve for a or for b	M1	
		Obtian $a = 7$ and $b = 4$	A1	[5]
	(ii)	*EITHER*: State or imply $(x + 2)^2$ is a factor	B1	
		Attempt division by $(x + 2)^2$ reaching a quotient $2x + k$ or use inspection with unknown factor $cx + d$ reaching $c = 2$ or $d = -1$	M1	
		Obtain factorisation $(x + 2)^2 (2x - 1)$	A1	
		OR: Attempt division by $(x + 2)$	M1	
		Obtain quadratic factor $2x^2 + 3x - 2$	A1	
		Obtain factorisation $(x + 2)(x + 2)(2x - 1)$	A1	[3]
		[The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + ex + f$ and an equation in e and/or f, or if two coefficients with the correct moduli are stated without working.]		

6	**(i)** State or imply $\dfrac{dx}{d\theta} = 2\sec^2\theta$ or $dx = 2\sec^2\theta\, d\theta$	B1	
	Substitute for x and dx throughout	M1	
	Obtain any correct form in terms of θ	A1	
	Obtain the given form correctly (including the limits)	A1	[4]
	(ii) Use cos 2A formula, replacing integrand by $a + b\cos 2\theta$, where $ab \neq 0$	M1*	
	Integrate and obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	A1	
	Use limits $\theta = 0$ and $\theta = \frac{1}{4}\pi$	M1(dep*)	
	Obtain answer $\frac{1}{8}(\pi + 2)$, or exact quivalent	A1	[4]
7	**(i) (a)** State that $u + v$ is equal to $1 + 2\mathrm{i}$	B1	[1]
	(b) *EITHER*: Multiply numerator and denominator of u/v by $3 - \mathrm{i}$, or equivalent	M1	
	Simplify numerator to $-5 + 5\mathrm{i}$, or denominator to 10	A1	
	Obtain answer $-\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
	OR1: Obtain two equations in x and y and solve for x or for y	M1	
	Obtain $x = -\frac{1}{2}$ or $y = \frac{1}{2}$	A1	
	Obtain answer $-\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
	OR2: Using the correct processes express u/v in polar form	M1	
	Obtain $x = -\frac{1}{2}$ or $y = \frac{1}{2}$ correctly	A1	
	Obtain answer $-\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	[3]
	(ii) State that the argument of u/v is $\frac{3}{4}\pi$ (2.36 radians or 135°)	B1√	[1]
	(iii) *EITHER*: Use facts that angle $AOB = \arg u - \arg v$ and $\arg u - \arg v = \arg(u/v)$	M1	
	Obtain given answer	A1	
	OR1: Obtain tan $A\hat{O}B$ from gradients of OA and OB and the tan $(A \pm B)$ formula	M1	
	Obtain given answer	A1	
	OR2: Obtain cos $A\hat{O}B$ by using the cosine formula or scalar product	M1	
	Obtain given answer	A1	[2]
	(iv) State $OA = BC$	B1	
	State OA is parallel to BC	B1	[2]
8	**(i)** State or imply partial fractions are of the form $\dfrac{A}{1-x} + \dfrac{Bx + C}{2 + x^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain $A = \frac{2}{3}$, $B = \frac{2}{3}$ and $C = \frac{1}{3}$	A1 + A1 + A1	[5]

(ii) Use correct method to find first two terms of the expansion of $(1-x)^{-1}$, $(2+x^2)^{-1}$ or $(1+\frac{1}{2}x^2)^{-1}$ — M1

Obtain complete unsimplified expansions up to x^2 of each partial fraction e.g. $\frac{2}{3}(1+x+x^2)$ and $\frac{1}{2}(\frac{2}{3}x-\frac{1}{3})(1-\frac{1}{2}x^2)$ — A1√ + A1√

Carry out multiplication of $(2+x^2)^{-1}$ by $(\frac{2}{3}x-\frac{1}{3})$, or equivalent, provided $BC \neq 0$ — M1

Obtain answer $\frac{1}{2}+x+\frac{3}{4}x^2$ — A1 [5]

[Symbolic binomial coefficients are not sufficient for the first M1. The f.t. is on A, B, C.]
[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10]
[In the case of an attempt to expand $(1+x)(1-x)^{-1}(2+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]
[Allow Maclaurin, giving M1A1√A1√ for differentiating and obtaining $f(0)=\frac{1}{2}$ and $f'(0)=1$, A1√ for $f''(0)=\frac{3}{2}$, and A1 for the final answer (the f.t. is on A, B, C if used).]

9 **(i)** Separate variables correctly — B1
Integrate and obtain term $\ln(\theta - A)$, or equivalent — B1
Integrate and obtain term $-kt$, or equivalent — B1
Use $\theta = 4A$, $t = 0$ to determine a constant, or as limits — M1
Obtain correct answer in any form, e.g. $\ln(\theta - A) = -kt + \ln 3A$, with no errors seen — A1 [5]

(ii) Substitute $\theta = 3A$, $t = 1$ and justify the given statement — B1 [1]

(iii) Substitute $t = 2$ and solve for θ in terms of A — M1
Remove logarithms — M1
Obtain answer $\theta = \frac{7}{3}A$, or equivalent, with no errors seen — A1 [3]
[The M marks are only available if the solution to part **(i)** contains terms $a\ln(\theta - A)$ and bt.]

10 **(i)** Substitute coordinates (1, 4, 2) in $2x - 3y + 6z = d$ — M1
Obtain plane equation $2x - 3y + 6z = 2$, or equivalent — A1 [2]

(ii) *EITHER*: Attempt to use plane perpendicular formula to find perpendicular from (1, 4, 2) to p — M1
Obtain a correct unsimplified expression, e.g. $\dfrac{|2-3(4)+6(2)-16|}{\sqrt{(2^2+(-3)^2+6^2)}}$ — A1
Obtain answer 2 — A1

OR1: State or imply perpendicular from O to p is $\frac{16}{7}$, or from O to q is $\frac{2}{7}$, or equivalent — B1
Find difference in perpendiculars — M1
Obtain answer 2 — A1

OR2: Obtain correct parameter value, or position vector or coordinates of foot of perpendicular from (1, 4, 2) to p ($\mu = \pm\frac{2}{7}$; $(\frac{11}{7}, \frac{22}{7}, \frac{26}{7})$) — B1
Calculate the length of the perpendicular — M1
Obtain answer 2 — A1

	OR3:	Carry out correct method for finding the projection onto a normal vector of a line segment joining a point on p, e.g. (8, 0, 0) and a point on q, e.g. (1, 4, 2)	M1	
		Obtain a correct unsimplified expression, e.g. $\dfrac{\lvert 2(8-1)-3(-4)+6(-2)\rvert}{\sqrt{(2^2+(-3)^2+6^2)}}$	A1	
		Obtain answer 2	A1	[3]
(iii)	*EITHER*:	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use scalar product to obtain a relevant equation in a, b and c	M1*	
		Obtain two correct equations, e.g. $2a - 3b + 6c = 0$, $a - 2b + 2c = 0$	A1	
		Solve for one ratio, e.g. $a : b$	M1(dep*)	
		Obtain $a : b : c = 6 : 2 : -1$, or equivalent	A1	
		State answer $\mathbf{r} = \lambda(6\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ or equivalent	A1√	
	OR:	Attempt to calculate vector product of two normals, e.g. $(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \times (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M2	
		Obtain two correct components	A1	
		Obtain $-6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, or equivalent	A1	
		State answer $\mathbf{r} = \lambda(-6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1√	[5]